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CONSTRUCTION OF THE CREEP EQUATIONS FOR MATERIALS WITH DIFFERENT EXTENSION AND COMPRESSION PROPERTIES

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UDC 539.376

Constructional materials of light alloys such as aluminum-magnesium and titanium possess different tensile properties for tension and compression. Whereas the "instantaneous" elastoplastic properties may differ only slightly, the difference in the properties under prolonged action (e.g., the duration up to fracture) may reach several orders of magnitude [1]. Figure 1 shows a diagram of the creep of VT-9 titanium alloy at a temperature of 400°C with different combinations of tension (compression) and twisting at a constant stress

$$\sigma_i = (\sigma^2 + 3\tau^2)^{1/2} = 72.5 \text{ kg/mm}^2$$

in the form of the time-dependence $A = \sigma\varepsilon + \tau\gamma$. The marks on the diagrams correspond to the marks of the scheme of the stressed state of the plane $\sigma - \sqrt{3}\tau$. It can be seen from the diagram that the intensity of the creep process with σ_i = const decreases as the stress state changes from pure tension to pure shear and compression. Here for comparison we show two diagrams, namely, pure tension with $\sigma_i = 71$ kg/mm^2 (points 1) and twisting of a thin-walled tubular specimen with $\sigma_i = 77.5$ kg/mm^2 (points 2), the intensity of the creep process of which is the same.* The example given clearly illustrates the need to construct a theory which would enable one to describe creep in complex media.

One of the first attempts to describe creep in media with different resistance to tension and compression is described in [2], in which the actual stresses are replaced by "reduced" stresses, and a theory is constructed assuming similarity between the deviators of the rates of deformation and the "reduced" stresses. This method has not been developed any further, and in practice even simple problems lead to quite complicated equations [3].

Another approach is to construct creep equations in the form of a dependence of the "equivalent rate of deformation" η_e on the "equivalent stress" σ_e, where the intensity of the rate of deformation $\eta_i = ({}^2/_3\eta_{kl}\eta_{kl})^{1/2}$ is usually taken as η_e, while σ_e is considered as a function of the stress tensor invariants. The creep equation is supplemented by the law of flow (e.g., by the gradient $\eta_{kl} = k\partial\sigma_e'/\partial\sigma_{kl}$, where σ_e' is not always the same as σ_e) [4, 5].

Attempts have been to construct equations assuming the existence of a potential creep function which depends on the stress tensor invariants and scalar parameters of the strengths [6-13]. The potential function is assumed to be both smooth [6-8, 12] and piecewise-smooth [9, 11, 13], and equations have also been constructed with more general assumptions [14].

When constructing defining equations the different resistance to tension and compression is taken into account by introducing into σ_e, in addition to the second invariant of the stress deviator, one of the odd invariants: In a number of papers preference is given to the first invariant of the stress tensor [6, 9, 11, 15, 16-18], while in others preference is given to the third invariant of the stress deviator [7, 8, 12, 14]. Although it is not our purpose to make a more detailed review of the papers in this field, we will illustrate the most typical approaches to constructing defining equations containing in addition either the first or third invariants of the stress tensor by using the example of the creep of OT-4 titanium alloy at a temperature of 475°C and different combinations of tension-twisting and compression-twisting.

* N. G. Torshenov participated in these experiments.

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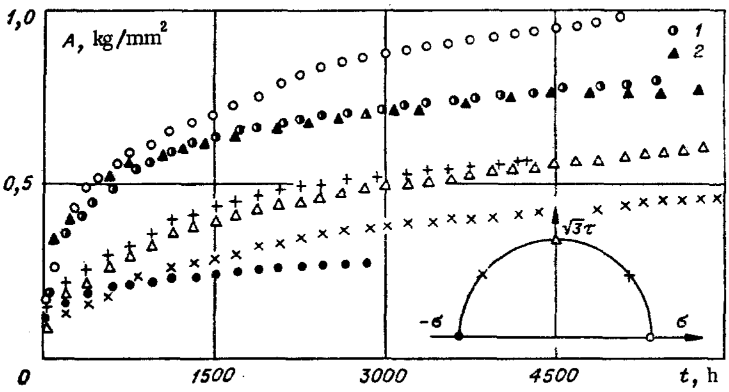

Fig. 1

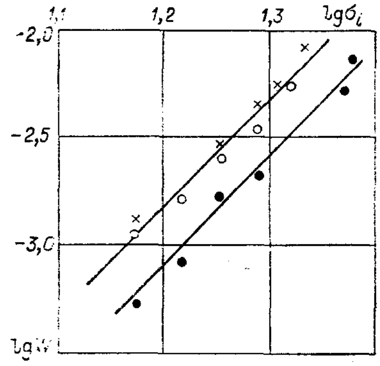

Fig. 2

The experimental program was carried out on specimens of the same geometrical shape and dimensions in order to eliminate the possible effect of scale factors. Tubular specimens with external and internal diameters of 20 mm and 18 mm and a length of the working part of 30 mm were made from a single bar 25 mm in diameter of OT-4 alloy. The specimens were not subjected to any thermal processing after manufacture. The anisotropic properties of the material were not checked. When analyzing the experimental data we gave particular attention to the difference in the properties of the material in tension and compression. The experiments were carried out on three machines in which the force and temperature modes of operation were first checked and compared. During the experiment from the condition of incompressibility of the material $F_0 l_0 = Fl$ at each 0.5% axial deformation of the creep we calculated the area of cross section of the specimen F and the mean radius of cross section $r = r_0 + \Delta r$ (here F_0 and F, l_0 and l are the initial and current cross-sectional area of the specimen and the lengths of its operating part, $\Delta r = r_0 \varepsilon_\varphi = -r_0 \varepsilon_z/2$) and from them we corrected the axial load and the torque in order to maintain the stressed state in the specimen constant. In the experiments on pure tension and also for combinations of tension and a small addition of torque the specimens were subjected to fracture. For other combinations of the stressed state the experiment was discontinued when indications appeared of a loss of stability in the specimen.

We will assume that the creep in the material in the steady-state stage with uniaxial loading can be described by the relation

$$\sigma\eta = B|\sigma|^n, \tag{1}$$

which when generalized to the case of a spatial stressed state leads to the relation

$$W = B_0 \sigma_e^n, \quad W = \sigma_{ij}\eta_{ij}. \tag{2}$$

Figure 2 shows in logarithmic coordinates the results of experiments on pure tension (the circles), pure compression (the dark points), and pure twisting (the crosses); it can be seen that the index n for all three forms of loading is the same, but σ_e is not the same as the intensity of these stresses σ_i. By processing the results of these experiments using a relation of the form (1) we obtain for the tension and compression, respectively,

$$B_1 = 13.3 \cdot 10^{-10} (\text{kg/mm}^2)^{1-n} \text{h}^{-1}, \; B_2 = 7.5 \cdot 10^{-10} (\text{kg/mm}^2)^{1-n} \text{h}^{-1}, \; n = 5. \tag{3}$$

The creep equation (2), which contains in addition to the second invariant of the stress deviator (the stress intensity) $\sigma_i = ((3/2)\sigma^0_{kl}\sigma^0_{kl})^{1/2}$, where $\sigma^0_{kl} = \sigma^0_{kl} - {}^1/_3 \sigma_{nn}\delta_{kl}$, the first invariant of the stress tensor $I_1 = \sigma_{nn}$, also, can, e.g., be represented in the form

$$W = \frac{1}{2}[B_1(1 + I_1/\sigma_i) + B_2(1 - I_1/\sigma_i)]\sigma_i^n, \tag{4}$$
$$\eta_{kl} = k_1 \frac{\partial \sigma_i}{\partial \sigma_{kl}}$$

with the additional limitation $-1 \leq I_1/\sigma_i \leq 1$ and $|I_1/\sigma_i| \equiv 1$ outside this interval.

In the space of the principal stresses the surface W = const according to (4) represents two components equally inclined to the coordinate axes of the cylinder with a transition region. When the difference in the properties for tension and compression is reduced relation (4) reduces to the well-known relation for isotropic media in accordance with Mises criterion with the associated flow law. In Fig. 3 the number 1 represents the contour W = const in the plane $\sigma_1 - \sigma_2$, constructed from relation (4) with the characteristics (3).

TABLE 1

Experiment No.	σ, $\frac{kg}{mm^2}$	τ, $\frac{kg}{mm^2}$	$\eta_e \cdot 10^4$	$\zeta_e \cdot 10^4$	$\eta_1 \cdot 10^4$	$\zeta_1 \cdot 10^4$	$\eta_2 \cdot 10^4$	$\zeta_2 \cdot 10^4$	$\eta_3 \cdot 10^4$	$\zeta_3 \cdot 10^4$	$\eta_4 \cdot 10^4$	$\zeta_4 \cdot 10^4$
1	21	0	2,59	0	2,59	0	2,59	0	2,59	0	2,59	0
2	19,49	4,66	2,59	1,80	2,40	1,72	2,57	2,46	2,43	1,72	2,56	2,32
3	15,00	8,66	1,76	3,36	1,78	3,08	2,22	5,12	1,90	3,10	2.28	4,86
4	8,10	11,20	1,10	5,40	0,93	3,86	1,32	7,37	1,02	3,57	1,48	7,05
5	0	11,70	0	4,70	0	3,03	0,15	8,20	3,00	3,00	0,16	6,04
6	—8,40	11,70	—0,97	4,27	—0,82	3,44	—1,22	6,77	—0,63	3,28	—1.00	6,70
7	—15,90	9,18	—1,95	3,25	—1,51	2,62	—1,94	4,47	—1,36	2,52	—1.80	4,55
8	—24,00	0	—2,84	0	—2,48	0	—2,48	0	—2,49	0	—2,48	0
9	9,05	9,05	0,60	2,76	0,63	1,91	0,87	3,49	0,72	1,91	0,94	3,34
10	15,63	5,21	1,08	1,28	1,18	1,18	1,32	1,77	1,22	1,19	1.34	1,67
11	0	8,66	0	1,55	0	0,91	0	1,87	0,04	0,91	0,04	1,81
12	0	11,25	0	3,80	0	2,60	0	5,33	0,13	2,58	0.14	5,16
13	0	10,39	0	2,76	0	1,89	0	3,88	0,09	1,88	0,10	3,80
14	0	12,50	0	6,80	0	3,96	0	8,20	0,19	3,91	0,22	7,87

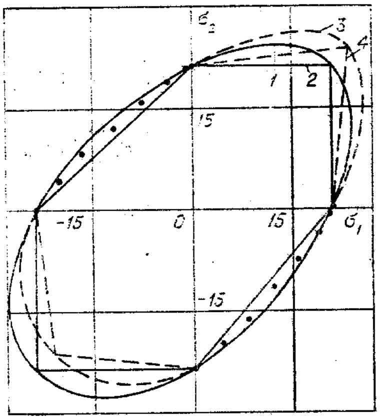

Fig. 3

In Table 1 we give experimental data of the rates of deformation of the creep $\eta = d\varepsilon/dt$ and $\zeta = d\gamma/dt$, corresponding to different combinations of σ and τ. In the columns η_1 and ζ_1 we show the calculated values of the rates of deformation obtained from Eq. (4). The points in Fig. 3 denote the experimental values of $W = \sigma\eta + \tau\gamma$ of the first seven experiments in Table 1.

By analogy with (4) we can give a generalization of the Tresk–St. Venant criterion in the form

$$W = \frac{1}{2}\left[B_1\left(1 + \frac{I_1}{\tau_m}\right) + B_2\left(1 - \frac{I_1}{\tau_m}\right)\right]\tau_m^n, \qquad (5)$$
$$\eta_j = k_2 \frac{\partial \tau_m}{\partial \sigma_j}, \; \tau_m = \sigma_1 - \sigma_3, \; \sigma_1 > \sigma_2 > \sigma_3$$

with the limitations $-1 \le I_1/\tau_m \le 1$ and $|I_1/\tau_m| \equiv 1$ outside this interval.

In the space of the principal stresses the surface W = const according to (5) represents two six-sided prisms equally inclined to the coordinate axes, connected by a pyramidal region. When B_1 approaches B_2 relations (5) reduce to the maximum tangential stress criterion with the associated flow law, well known for isotropic media. In Fig. 3 the number 2 denotes the contour W = const, constructed from relation (5) with the same characteristics (3). In columns η_2 and ζ_2 in Table 1 we show the calculated values of the rates of deformation of the creep obtained from these relations.

An example of the creep equation the arguments of which contain the second and third invariants of the stress deviator is

$$W = B[(1 + a\sin 3\xi)\,\sigma_i]^n, \; \eta_{ij} = k_3 \frac{\partial \sigma_э}{\partial \sigma_{ij}}, \; \sigma_e = (1 + a\,\sin 3\xi)\,\sigma_i, \qquad (6)$$

where

$$\sin 3\xi = -\frac{9}{2}\frac{\sigma^0_{kl}\sigma^0_{ln}\sigma^0_{nk}}{\sigma_i^3}, \; B = \frac{1}{2^n}\left(B_1^{1/n} + B_2^{1/n}\right)^n; \; a = \frac{B_2^{1/n} - B_1^{1/n}}{B_2^{1/n} + B_1^{1/n}}.$$

In the space of the principal stresses the surface W = const, according to relation (6), is a noncircular cylinder with an axis coinciding with the hydrostatic axis; the vector of the deformation velocity is orthogonal to the surface W = const. As the difference between B_1 and B_2 decreases relation (6) again reduces to the Mises criterion. In Fig. 3 the number 3 denotes the contour W = const constructed from relation (6) with characteristics (3), and the theoretical values of the rate of deformation from these relations are shown in columns η_3 and ζ_3 of Table 1.

An example of a piecewise-linear function of the defining equation which is a natural generalization of the maximum tangential stress criterion is the relation

$$W = B_1[\sigma_1 - (1 - \lambda)\sigma_2 - \lambda\sigma_3]^n, \; \sigma_1 > \sigma_2 > \sigma_3, \; \eta_j = k_4 \frac{\partial \sigma_e}{\partial \sigma_j}, \; \sigma_э = \sigma_1 - (1 - \lambda)\,\sigma_2 - \lambda\sigma_3, \; \lambda = \left(\frac{B_2}{B_1}\right)^{1/n}. \qquad (7)$$

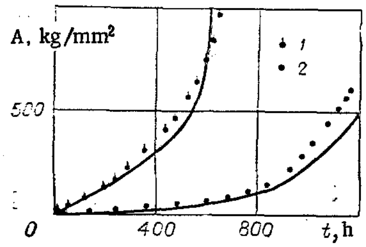

Fig. 4

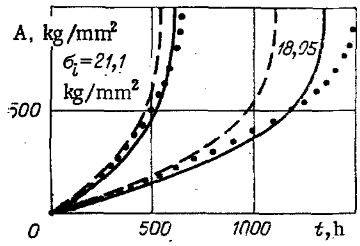

Fig. 5

In the space of the principal stresses the surface W = const in this case is an irregular prism. As B_1 approaches B_2 criterion (7) reduces to the maximum tangential stress criterion already mentioned above. In Fig. 3 the number 4 denotes the contour W = const corresponding to relation (7), and the calculated values of the deformation rates are shown in columns η_4 and ζ_4 of Table 1.

It can be seen from Fig. 3 and Table 1 that all the four types of relations (4)-(7) represented there in the region of the stressed states, corresponding to different combinations of tension (compression) –twisting, are close to one another, and it is impossible to establish the advantage of any of them from these experiments. Moreover, relations (4) and (6), which represent a generalization of the Mises criterion due to the introduction of either the first invariant I_1 or the third invariant S_3 respectively, give practically the same results for this region of the stressed states. The same applies to relations (5) and (7), which generalize the maximum shear criterion in a similar way. We would expect a big difference between them in regions of other stress states.

The relations $W = \varphi(\sigma_e)$ represented above, describing the steady-state creep processes, can also be generalized to the case of softening. We will assume, as in the case of an isotropic medium [19], that the damage to the material leading to its softening is directly proportional to the value of the work dissipated in irreversible deformations $A = \int_0^t \sigma_{ij}\eta_{ij}\,dt$. Using the methods developed in [19] we will write any of the relations (4)-(7) in the form

$$W = \frac{A_*^m \varphi\,(\sigma_e)}{(A_* - A)^m} \tag{8}$$

with the addition of the flow law corresponding to each of the relations (4)-(7). For example, as it applies to relation (4) we have

$$\varphi(\sigma_e) = \frac{1}{2}\,[B_1(1 + I_1/\sigma_i) + B_2(1 - I_1/\sigma_i)]\,\sigma_i^n.$$

In particular, for pure tension and pure compression we obtain

$$W = \frac{A_*^m B_1 \sigma^n}{(A_* - A)^m} \text{ and } W = \frac{A_*^m B_2\,|\,\sigma\,|^n}{(A_* - A)^m}. \tag{9}$$

In Fig. 4 the points 1 represent the results of experiment on creep to fracture for pure tension with σ = 21 kg/mm², from which, using the method described in [19], we determine the deficient characteristics (assuming that they are independent of the form of the stressed state)

$$A_* = 10 \text{ kg/mm}^2,\ m = 2. \tag{10}$$

The continuous line represents the theoretical diagram from the first of relations (9), and points 2 represent the results of experiment for pure compression with overloading. The values of the compression stresses σ kg/mm² and the duration of the action Δt_h of each of them are given in Table 2. The continuous line represents the calculated diagram obtained from the second of relations (9), with the same characteristics (10), obtained from experiments on tension.

In Fig. 5 the points represent the results of two experiments on tension with twisting for σ_i = 21.1 kg/mm² (σ = 19.49 kg/mm², and τ = 4.66 kg/mm²) and σ_i = 18.05 kg/mm² (σ = 15.63 kg/mm², and τ = 5.21 kg/mm²). The ratios of the creep deformation increments were kept unchanged during the experiment practically up to fracture, and had the values $\Delta\varepsilon/\Delta\gamma$ = 1.48 for the first and $\Delta\varepsilon/\Delta\gamma$ = 1.1 for the second, which is quite close to the corresponding values of the ratios $\sigma/3\tau$. The continuous and dashed curves show the theoretical diagrams from (8) with $\varphi(\sigma_e)$ specified by relations (6) and (7), respectively.

TABLE 2

Δt, h	184	262	226	134	63	300
σ, kg/mm²	15	16,5	18	19,5	21	23,5

It follows from Figs. 4 and 5 that as in the case of the creep of isotropic media taking softening and fracture into account, in media with different properties for tension and compression the defining equation (8) can be written in the form $W = \varphi(\sigma_e)\psi(A)$, where all the differences in the behavior of the material are reflected in the function $\varphi(\sigma_e)$.

The quantity $A = \int_0^t \sigma_{ij}\eta_{ij}\,dt$ is an isotropic softening parameter which with sufficient accuracy for practical purposes can be assumed to be independent of the shape of the stressed state and can be found, as in the case of an isotropic material, by experiments on stretching.

All the relations (4)-(7) given above and the corresponding generalizations to the case of softening were constructed taking into account the characteristics of the material obtained only from experiments on pure tension or pure compression. It can be seen that these relations assume a monotonic increase in the values of σ_i or τ_m when the stressed state changes from pure tension to twisting, and further to compression along the contour W = const. Otherwise when σ_i = const the intensity of the process in stretching W^+ will be greater than in twisting W^0, and greater than in compression, i.e., it is assumed that the situation represented in Fig. 1 for VT-9 material is realized. In fact the inequality $W^+ > W^0 > W^-$ is not always satisfied. Thus, it can be seen from Fig. 2 that for OT-4 material creep is estimated in the form of the inequality $W^+ = W^0 > W^-$ for the same values of σ_i in tension, twisting, and compression. We can give an example when the values of the processes during shear W^0 even lie outside the range of W^- and W^+, i.e., we have $W^- < W^+ < W^0$ [14]. This fact is obviously a specific feature of materials possessing different properties for tension and compression: The shear characteristics of the material must be regarded as independent "certificate data" and the defining equations must be constructed on the basis of three characteristics: tension, compression, and shear. In this sense relations (4)-(7) given above must be regarded as a first approximation.

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LOWER LIMIT TO THE STRENGTH OF SURFACE FORCES IN THE CASE OF PLANE STRAIN OF AN IDEAL RIGID-PLASTIC MEDIUM

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A lower limit to the strength of surface forces based on the use of a statically permissible stress field follows from the extremum theorems of an ideal rigid-plastic medium [1]. It is also known that the stress field in a rigid-plastic medium with a convex plasticity condition is unique in those zones in which the deformation rates are different from zero [2]. It is shown in this paper that there exists for the class of problems in which a functional corresponding to the lower limit of the strength of the external surface forces is nonidentically equal to a constant on a set of statically permissible stress fields a stress field which yields a maximum of this functional.

1. Let Ω be a region with a piecewise-continuous boundary S on the (x, y) plane, and let $\text{mes}(\Omega) < \infty$. A stress field $(\sigma_x, \sigma_y, \tau)$ which is continuous and continuously differentiable satisfies the equilibrium conditions in Ω

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial y} + f_x = 0, \ \frac{\partial \tau}{\partial x} + \frac{\partial \sigma_y}{\partial y} + f_y = 0, \tag{1.1}$$

and the boundary conditions on part of the boundary S_σ

$$\begin{aligned} \sigma_n &= \sigma_x n_x^2 + \sigma_y n_y^2 + 2\tau n_x n_y = g(S), \\ \tau_n &= (\sigma_y - \sigma_x) n_x n_y + \tau (n_x^2 - n_y^2) = h(S) \end{aligned} \tag{1.2}$$

and does not violate the plasticity condition in $\bar{\Omega} = \Omega + S$,

$$\frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau^2 \leqslant \tau_s^2 \tag{1.3}$$

is called statically permissible.

A velocity field (u, v) which satisfies the incompressibility condition in Ω

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1.4}$$

and the boundary conditions on the part of the boundary $S_u = S - S_\sigma$

$$u = u_0(S), \ v = v_0(S) \tag{1.5}$$

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